Q 5.1) Answer the following:

(a) A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field.

(b) The angle of dip at a location in southern India is about 18°. Would you expect a greater or smaller dip angle in Britain?

(c)If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground?

(d) In which direction would a compass free to move in the vertical plane point to, if located right on the geomagnetic north or south pole?

(e) The earth's field, it is claimed, roughly approximates the field due to a dipole of the magnetic moment $8 \times \ 1022 \ J \ T^{-1}$ **located at its centre. Check the order of magnitude of this number in some way.**

(f) Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all?

Answer 5.1:

(a) The three independent conventional quantities used for determining the earth's magnetic field are:

(i) Magnetic declination,

(ii) Angle of dip

(iii) Horizontal component of earth's magnetic field

(b) The angle of dip at a point depends on how far the point is located with respect to the North Pole or the South Pole. Hence, as the location of Britain on the globe is closer to the magnetic North pole, the angle of dip would be greater in Britain (About 70°) than in southern India.

(c) It is assumed that a huge bar magnet is submerged inside the earth with its north pole near the geographic South Pole and its south pole near the geographic North Pole.

Magnetic field lines originate from the magnetic north pole and terminate at the magnetic south pole. Hence, in a map depicting earth's magnetic field lines, the field lines at Melbourne, Australia would seem to move away from the ground.

(d) If a compass is placed in the geomagnetic North Pole or the South Pole, then the compass will be free to move in the horizontal plane while the earth's field is exactly vertical to the magnetic poles. In such a case, the compass can point in any direction.

(e) Magnetic moment, M = $8 \times \ 10^{22} \ J \ T^{-1}$

Radius of earth, r = $6.4 \times \ 10^6 \ m$

Magnetic field strength, B = $\frac{\mu_0 M}{4\pi r^3}$

Where,

μ_0 = Permeability of free space = $4\pi \times \ 10^{-7} \ TmA^{-1}$

Therefore, B = $\frac{4\pi\times10^{-7}\times8\times10^{22}}{4\pi\times(6.4\times10^6)^3} = 0.3 \ G$

This quantity is of the order of magnitude of the observed field on earth.

(f) Yes, there are several local poles on earth's surface oriented in different directions. A magnetized mineral deposit is an example of a local N-S pole.

Q 5.2) Answer the following:

(a) The earth's magnetic field varies from point to point in space. Does it also change with time? If so, on what time scale does it change

appreciably?

(b) The earth's core is known to contain iron. Yet geologists do not regard this as a source of the earth's magnetism. Why?

(c) The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the 'battery' (i.e., the source of energy) to sustain these currents?

(d) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?

(e) The earth's field departs from its dipole shape substantially at large distances (greater than about 30,000 km). What agencies may be responsible for this distortion?

(f) Interstellar space has an extremely weak magnetic field of the order of 10^{-12} T. Can such a weak field be of any significant consequence? Explain.

Answer 5.2:

(a) Earth's magnetic field varies with time and it takes a couple of hundred years to change by an obvious sum. The variation in the Earth's magnetic field with respect to time can't be ignored.

(b) The Iron core at the Earth's centre cannot be considered as a source of Earth's magnetism because it is in its molten form and is non-ferromagnetic.

(c) The radioactivity in earth's interior is the source of energy that sustains the currents in the outer conducting regions of earth's core. These charged currents are considered to be responsible for earth's magnetism.

(d) The Earth's magnetic field reversal has been recorded several times in the past about 4 to 5 billion years ago. These changing magnetic fields were weakly recorded in rocks during their solidification. One can obtain clues about the geomagnetic history from the analysis of this rock magnetism.

(e) Due to the presence of ionosphere, the Earth's field deviates from its dipole shape substantially at large distances. The Eart's field is slightly modified in this region because of the field of single ions. The magnetic field associated with them is produced while in motion.

(f) A remarkably weak magnetic field can deflect charged particles moving in a circle. This may not be detectable for a large radius path. With reference to the gigantic interstellar space, the deflection can alter the passage of charged particles.

Q 5.3) A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.25 T experiences a torque of magnitude equal to $4.5 \times 10^{-2}\ J$. What is the magnitude of the magnetic moment of the magnet?

Answer 5.3:

Magnetic field strength, B = 0.25 T

Torque on the bar magnet, T = $4.5 \times 10^{-2}\ J$

The angle between the bar magnet and the external magnetic field, $\theta = 30^\circ$

Torque is related to magnetic moment (M) as:

$$T = MBsin\theta \ \therefore \ M = \frac{T}{Bsin\theta}$$

= $\frac{4.5 \times 10^{-2}}{0.25 \times sin30^\circ} = 0.36\ J\,T^{-1}$

Hence, the magnetic moment of the magnet is $0.36\ J\,T^{-1}$.

Q 5.4) A short bar magnet of magnetic moment m = 0.32 JT^{-1} is placed in a uniform magnetic field of 0.15 T. If the bar is free to rotate in the plane of the field, which orientation would correspond to its (a) stable, and (b) unstable equilibrium? What is the potential energy of the magnet in each case?

Answer 5.4:

Moment of the bar magnet, M = $0.32\ J\,T^{-1}$

External magnetic field, B = 0.15 T

(a) The bar magnet is aligned along the magnetic field. This system is considered as being in stable equilibrium. Hence, the angle θ,

between the bar magnet and the magnetic field is 0°.

Potential energy of the system = $-MBcos\theta$

= $-0.32 \times\ 0.15\ cos0^\circ$

= $-4.8 \times\ 10^{-2}\ J$

(b) The bar magnet is oriented 180° to the magnetic field. Hence, it is in unstable equilibrium. $\theta =\ 180^\circ$

Potential energy = $-MBcos\theta$

= $-0.32 \times\ 0.15\ cos180^\circ$

= $4.8 \times\ 10^{-2}\ J$

Q 5.5) A closely wound solenoid of 800 turns and area of cross-section 2.5 × 10^{-4} m^2 carries a current of 3.0 A. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment?

Answer 5.5:

Number of turns in the solenoid, n = 800

Area of cross-section, A = $2.5 \times\ 10^{-4}\ m^2$

Current in the solenoid, I = 3.0 A

A current-carrying solenoid behaves like a bar magnet because a magnetic field develops along its axis, i.e., along with its length.

The magnetic moment associated with the given current-carrying solenoid is calculated as:

M = n I A

= $800 \times\ 3 \times\ 2.5 \times\ 10^{-4}$

= $0.6\ J\ T^{-1}$

Q 5.6) If the solenoid in Exercise 5.5 is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of the torque on the solenoid when its axis makes an angle of 30° with the direction of applied field?

Answer 5.6:

Magnetic field strength, B = 0.25 T

Magnetic moment, M = 0.6 T^{-1}

The angle θ, between the axis of the solenoid and the direction of the applied field is 30°.

Therefore, the torque acting on the solenoid is given as:

$\tau = MBsin\theta$

= $0.6times0.25\ sin30^{\circ}$

= $7.5 \times\ 10^{2}$ J

Q 5.7) A bar magnet of magnetic moment 1.5 J T^{-1} lies aligned with the direction of a uniform magnetic field of 0.22 T.

(a) What is the amount of work required by an external torque to turn the magnet so as to align its magnetic moment: (i) normal to the field direction, (ii) opposite to the field direction?

(b) What is the torque on the magnet in cases (i) and (ii)?

Answer 5.7:

(a) Magnetic moment, M = $1.5\ J\ T^{-1}$

Magnetic field strength, B = 0.22 T

(i) Initial angle between the axis and the magnetic field, $\theta_1\ =\ 0^{\circ}$

Final angle between the axis and the magnetic field, $\theta_2\ =\ 90^{\circ}$

The work required to make the magnetic moment normal to the direction of magnetic field is given as:

W = $-MB(cos\theta_2 - cos\theta_1)$

= $-1.5 \times\ 0.22(cos90^{\circ} - cos0^{\circ}$

= − 0.33 (0 − 1)

= 0.33 J

(ii) Initial angle between the axis and the magnetic field, $\theta_1\ =\ 0^{\circ}$

Final angle between the axis and the magnetic field, $\theta_2\ =\ 180^{\circ}$

The work required to make the magnetic moment opposite to the direction of magnetic field is given as:

W = $-MB(cos\theta_2 - cos\theta_1)$

= $-1.5 \times\ 0.22(cos180^{\circ} - cos0^{\circ})$

= − 0.33 (− 1 − 1)

= 0.66 J

(b) For case (i): $\theta\ =\ \theta_2\ =\ 90^{\circ}\ \therefore$ Torque, $\tau\ =\ MBsin\theta$

= $MBsin90^{\circ}$

= 0.33 J

For case (ii): $\theta = \theta_2 = 180^{\circ}$ $\therefore$ Torque, $\tau = MBsin\theta$

= $MBsin180^{\circ}$

= 0 J

Q 5.8) A closely wound solenoid of 2000 turns and area of cross-section $1.6 \times 10^{-4}\ m^2$, carrying 4.0 A current, is suspended through its centre, thereby allowing it to turn in a horizontal plane.

(a) What is the magnetic moment associated with the solenoid?

(b) What is the force and torque on the solenoid if a uniform the horizontal magnetic field of 7.5×10^{-2} T is set up at an angle of 30° with the axis of the solenoid?

Answer 5.8:

Number of turns on the solenoid, n = 2000

Area of cross-section of the solenoid, A = $1.6 \times 10^{-4}\ m^2$

Current in the solenoid, I = 4.0 A

(a) The magnetic moment along the axis of the solenoid is calculated as:

M = nAI

= $2000 \times 4 \times 1.6 \times 10^{-4}$

= 1.28 Am^2

(b) Magnetic field, B = $7.5 \times 10^{-2}\ T$

Angle between the magnetic field and the axis of the solenoid, $\theta = 30^{\circ}$

Torque, $\tau = MBsin\theta$

= $1.28 \times 7.5 \times 10^{-2}\ sin30^{\circ}$

= $0.048J$

Since the magnetic field is uniform, the force on the solenoid is zero. The torque on the solenoid is $0.048J$.

Q 5.9) A circular coil of 16 turns and radius 10 cm carrying a current of 0.75 A rests with its plane normal to an external field of magnitude 5.0 × 10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of 2.0 s^{-1}. What is the moment of inertia of the coil about its axis of rotation?

Answer 5.9:

Number of turns in the circular coil, N = 16

Radius of the coil, r = 10 cm = 0.1 m

Cross-section of the coil, A = $\pi r^2 = n \times (0.1)^2\ m^2$

Current in the coil, I = 0.75 A

Magnetic field strength, B = $5.0 \times 10^{-2}\ T$

Frequency of oscillations of the coil, v = $2.0\ s^{-1}$ $\therefore$ Magnetic moment, M = NIA = $NI\pi r^2$

= $16 \times 0.75 \times n \times (0.1)^2$

= $0.377\ J\ T^{-1}$

Frequency is given by the relation:

$$v = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}$$

Where,

I = Moment of inertia of the coil

$\therefore\ I = \frac{MB}{4\pi^2 v^2}$

= $\frac{0.377 \times 5 \times 10^{-2}}{4\pi^2 \times (2)^2}$

= $1.19 \times 10^{-4}\ kg\ m^2$

Hence, the moment of inertia of the coil about its axis of rotation is $1.19 \times 10^{-4}\ kg\ m^2$.

Q 5.10) A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at 22° with the horizontal. The horizontal component of the earth's magnetic field at the place is known to be 0.35 G. Determine the magnitude of the earth's magnetic field at the place.

Answer 5.10:

Horizontal component of earth's magnetic field, $B_H = 0.35\ G$

Angle made by the needle with the horizontal plane = Angle of dip = $\delta = 22^{\circ}$

Earth's magnetic field strength = B

We can relate B and B_H as:

$$B_H = B\cos\delta \therefore B = \frac{B_H}{\cos\delta}$$

= $\frac{0.35}{\cos 22^{\circ}}$ = 0.377 G

Hence, the strength of earth's magnetic field at the given location is 0.377 G.

Q 5.11) At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.

Answer 5.11:

Angle of declination, $\theta = 12^\circ$

Angle of dip, $\delta = 60^\circ$

Horizontal component of earth's magnetic field, $B_H = 0.16\ G$

Earth's magnetic field at the given location = B

We can relate B and B_H as:

$$B_H = Bcos\delta \therefore B = \frac{B_H}{cos\delta}$$

= $\frac{0.16}{cos\ 60^\circ}$ = 0.32 G

Earth's magnetic field lies in the vertical plane, 12° West of the geographic meridian, making an angle of 60° (upward) with the horizontal direction. Its magnitude is 0.32 G.

Q 5.12) A short bar magnet has a magnetic moment of 0.48 J T^{-1}. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on (a) the axis, (b) the equatorial lines (normal bisector) of the magnet.

Answer 5.12:

Magnetic moment of the bar magnet, M = $0.48\ J\ T^{-1}$

(a) Distance, d = 10 cm = 0.1 m

The magnetic field at distance d, from the centre of the magnet on the axis, is given by the relation:

$$B = \frac{\mu_0\ 2M}{4\pi d^3}$$

Where,

μ_0 = Permeability of free space = $4\pi \times\ 10^{-7}\ TmA^{-1} \therefore B = \frac{4\pi\times10^{-7}\times2\times0.48}{4\pi\times(0.1)^3}$

= $0.96 \times\ 10^{-4}\ T = 0.96\ G$

The magnetic field is along the S-N direction.

(b) The magnetic field at a distance of 10 cm (i.e., d = 0.1 m) on the equatorial line of the magnet is given as:

$$B = \frac{\mu_0\times M}{4\pi\times d^3}$$

= $\frac{4\pi\times10^{-7}\times0.48}{4\pi(0.1)^3}$

= 0.48 G

The magnetic field is along the N – S direction.

Q 5.13) A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic north-south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0.36 G and the angle of dip is zero. What is the total magnetic field on the normal bisector of the magnet at the same distance as the null–point (i.e., 14 cm) from the centre of the magnet? (At null points, field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field.)

Answer 5.13:

Earth's magnetic field at the given place, H = 0.36 G

The magnetic field at a distance d, on the axis of the magnet, is given as:

$$B_1 = \frac{\mu_0 2M}{4\pi d^3} = H \qquad \ldots(i)$$

Where,

μ_0 = Permeability of free space

M = Magnetic moment

The magnetic field at the same distance d, on the equatorial line of the magnet, is given as:

$$B_2 = \frac{\mu_0 M}{4\pi d^3} = \frac{H}{2} \qquad [Using\ equation\ (i)]$$

Total magnetic field, B = $B_1 + B_2$

= $H + \frac{H}{2}$

= 0.36 + 0.18 = 0.54 G

Hence, the magnetic field is 0.54 G in the direction of earth's magnetic field.

Q 5.14) If the bar magnet in exercise 5.13 is turned around by 180°, where will the new null points be located?

Answer 5.14:

The magnetic field on the axis of the magnet at a distance d_1 = 14 cm, can be written as:

$$B_1 = \frac{\mu_0 2M}{4\pi (d_1)^3} = H \qquad \ldots(1)$$

Where,

M = Magnetic moment

μ_0 = Permeability of free space

H = Horizontal component of the magnetic field at d_1

If the bar magnet is turned through , then the neutral point will lie on the equatorial line.

Hence, the magnetic field at a distance d_2 , on the equatorial line of the magnet can be written as:

$$B_1 = \frac{\mu_0 2M}{4\pi (d_1)^3} = H \qquad \ldots(2)$$

Equating equations (1) and (2), we get:

$\frac{2}{(d_1)^3} = \frac{1}{(d_2)^3} \left[\frac{d_2}{d_1}\right]^3 = \frac{1}{2} \therefore d_2 = d_1 \times \left(\frac{1}{2}\right)^{\frac{1}{3}}$

= $14 \times 0.794 = 11.1cm$

The new null points will be locked 11.1 cm on the normal bisector.

Q 5.15) A short bar magnet of magnetic moment 5.25 × 10^{-2} J T^{-1} is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45° with earth's field on (a) its normal bisector and (b) its axis. The magnitude of the earth's field at the place is given to be 0.42 G. Ignore the length of the magnet in comparison to the distances involved.

Answer 5.15:

Magnetic moment of the bar magnet, M = $5.25 \times 10^{-2}\ J\ T^{-1}$

Magnitude of earth's magnetic field at a place, H = 0.42 G = $0.42 \times 10^{-4}\ T$

(a) The magnetic field at a distance R from the centre of the magnet on the ordinary bisector is given by:

$B = \frac{\mu_0 M}{4\pi R^3}$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} TmA^{-1}$

When the resultant field is inclined at 45° with earth's field, B = H

$\therefore \frac{\mu_0 M}{4\pi R^3} = H = 0.42 \times 10^{-4}\ R^3 = \frac{\mu_0 M}{0.42 \times 10^{-4} \times 4\pi}$

= $R^3 = \frac{4\pi \times 10^{-7} \times 6.45 \times 10^{-2}}{4\pi \times 0.42 \times 10^{-4}}$

= $12.5 \times 10^{-5} \therefore R = 0.05\ m = 5\ cm$

(b) The magnetic field at a distanced 'R' from the centre of the magnet on its axis is given as:

$B' = \frac{\mu_0 2M}{4\pi R^3}$

The resultant field is inclined at 45° with the earth's field.

$\therefore B' = H\ \frac{\mu_0 2M}{4\pi (R')^3} = H\ (R')^3 = \frac{\mu_0\ 2M}{4\pi \times H}$

= $\frac{4\pi \times 10^{-7} \times 2 \times 5.25 \times 10^{-2}}{4\pi \times 0.42 \times 10^{-4}} = 25 \times 10^{-5} \therefore R = 0.063\ m = 6.3\ cm$